Optimization of an OTA Based Sine Waveshaper

openmusiclabs – February 2, 2017

I. Introduction

The most common analog Voltage Controlled Oscillator (VCO) cores are sawtooth and triangle wave generators. This is due to the simplicity and accuracy of charge integration with a single capacitor; converting a current into a frequency. These VCOs have rich harmonic content, and can be easily converted to square or pulse waves for even greater variation in timbre. But, there are times when a pure tone is desired, so conversion from the generated sawtooth or triangle wave is required. Sine wave based VCOs do exist, but are typically more complicated than their sawtooth counterparts, and as a result cost more and are more likely to have drift and accuracy issues. This paper briefly discusses some of the most popular ways of converting a triangle wave to a sine wave, and goes into depth on one the lowest distortion options, the Operational Transconductance Amplifier (OTA) with "cusp canceling".

II. Background

There are a number of ways to convert a triangle wave to a sine wave. Tracking filters can be use to remove the upper harmonics, but these need to be of high order (>4) or highly resonant as the harmonics are quite close to the fundamental. High order filters are large and expensive, and resonant filters need to track extremely well to keep the output amplitude constant. There are also some practical limitations to filtering at low enough frequencies for Low Frequency Oscillator (LFO) operations. Switched capacitor filters would do a very good job here, as they could be synchronized directly to the VCO output using a PLL, although lock times on the PLL could be quite long at low frequencies. Hybrid techniques employing a sine lookup table, Analog to Digital Converter (ADC), and Digital to Analog Converter (DAC) can be extremely accurate, low drift, and produce pure tones. They are versatile, in that alternate waveforms can be loaded into the lookup table, but are also more complicated and costly. For this work, we will be focusing on purely analog techniques, most of which being distortion based waveshapers.

A very good overview of waveshapers is given by Tim Stinchcombe on his website (http://www.timstinchcombe.co.uk/index.php?pge=trisin). Stinchcombe lists a number of good references, and to these we would like to add "Non-linear Circuits Handbook" from Analog Devices, Inc., 1976 (edited by Dan Sheingold), which has a number of different sine synthesis techniques, mostly using multipliers. We will use Stinchcombe's taxonomy for evaluating the various topologies: JFET, breakpoint, and differential pair.

A JFET waveshaper is shown in Figure 1, and is reported to give good results (<1% THD). Although this is a simple circuit, it has a number of temperature dependent components, so drift may be an issue. The distortion performance is also not as good as OTA methods which can achieve <0.05% THD. A thorough analysis of the circuit is warranted, as better performance may be attainable. Unfortunately, it is beyond the scope of this work, and will have to wait.

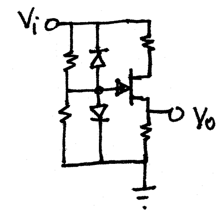

Fig. 1. Simple JFET based sine waveshaper.

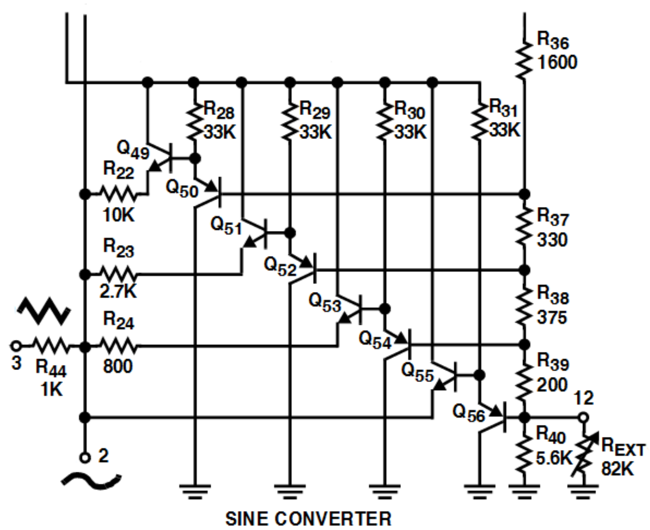

Fig. 2. Breakpoint waveshaping inside the ICL8030.

Breakpoint shaping involves setting a series of voltage levels at which the circuit gain is reduced. This was used inside of the ICL8038, and one half of its circuit is shown in Figure 2. Simpler methods are possible with diodes, but it's unclear if the complexity is worth it. The ICL8038 used 4 breakpoints, and only achieved a distortion of 1% after trimming. The XR8038 variant lists 0.3% THD, which is better, but still not very good compared to OTA methods. The main advantage to breakpoint shapers is that they don't rely as heavily on transistor parameters, and can therefore be more stable. For integrated circuits, where transistors are plentiful and well matched, a higher order shaper might make sense, but for discrete designs they are rather cumbersome.

Differential pair (or using an OTA) triangle to sine conversion provides the lowest distortion of the three presented options, and can be compact and inexpensive. But, it can also be hard to trim and has large temperature dependencies. There

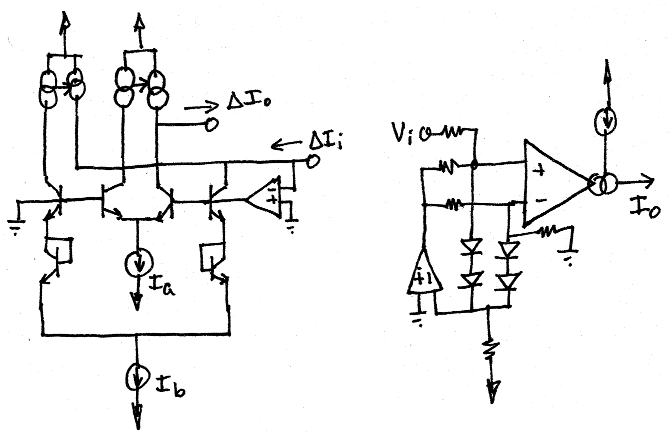

Fig. 3. Discrete and OTA versions of $x/(1-x^2)$ sine synthesis.

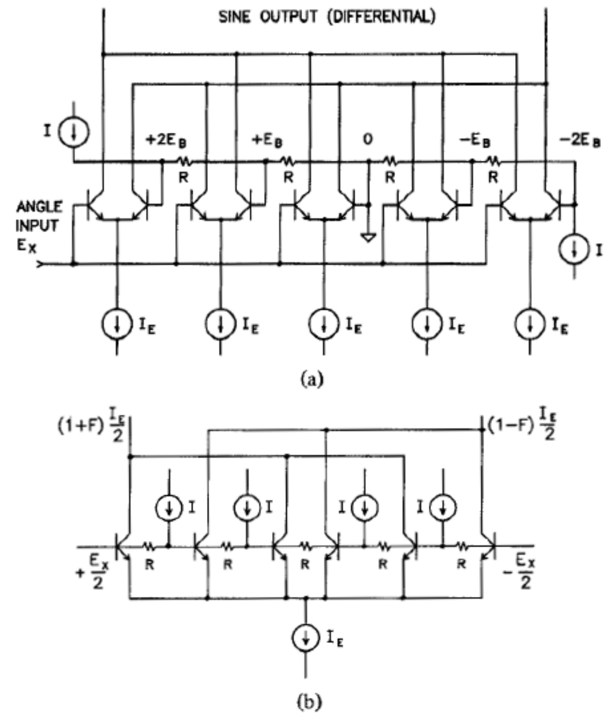

Fig. 4. Gilbert differential pair sine synthesis circuits.

are two main methods of OTA distortion: emitter degeneration and cusp canceling. The XR2206 used emitter degeneration, and claims 0.5% THD, but this can be greatly improved upon with cusp canceling. The remainder of this paper will present a mathematical model for the differential pair, compare degeneration to cusp canceling, and show distortion results for both techniques.

There are two other differential pair sine shapers worth mentioning. The first employs an $x/(1+x^2)$ circuit (Figure 3), and is shown in “Analog Circuit Design, Volume 4: Waveform Processing Ciruits” by Dennis Feucht, 2010 (originally stumbled across it here: http://m.eet.com/media/1051374/C0453pt2_4.pdf). The chapter is good, but Feucht erroneously identifies it as an x^2 circuit, in an attempt to synthesize the Taylor series directly. The higher order example he shows is also misidentified this way. In fact, it is better than an x^2 circuit, and is identical to the JFET based differential pair amplifier shown by Hassan in “FET Differential Amplifier as a Tri-Wave to Sine Converter”, 2004. This is due to the $I_D = I_{DSS}(1-(V_{gs}/V_p))^2$ transfer function of the JFET. As a result, the same theoretical minimum 0.005% distortion level can be achieved by either circuit, but the BJT version does not have temperature dependencies. Both circuits require either emitter degeneration or cusp canceling to achieve this low distortion value.

An even better version of a differential pair shaper is shown by Barrie Gilbert in “Circuits for the Precise Synthesis of the Sine Function”, 1977. Gilbert cascades several differential pair amplifiers at equally spaced breakpoints. In a way, it is similar to other breakpoint generators, but rather than attempting to adjust slope based on input voltage, it adds a series of approximate sines, with the errors of each canceling out. What is even more impressive, is that each breakpoint reverses the direction of the signal, so more than ±90° can be synthesized. The AD639 employed this method to achieve ±500° of sine generation with 0.02% THD over the first ±90°. The higher error in comparison to cusp canceling is due to the circuit being tuned for a wider range. If it is only to be used over the first ±90°, a simple four transistor variant can achieve the same theoretical 0.005% THD.

Gilbert gives multiple ways of implementing this circuit, and two of these are shown in Figure 4. The lower circuit achieves 180° per transistor, with a single current source for the tail currents. Another current source can be used for the bias circuitry, or a geometric resistor network can be used. The circuit has large temperature dependencies, but these can be canceled out with temperature dependent current sources. Ultimately, it is an incredibly useful circuit, but a bit complicated for basic sine shaping purposes.

III. OTA WAVE SHAPING ANALYSIS

To understand why the differential pair gives a good sine approximation, we can compare the Taylor series of both the differential amplifier transfer function and the sine function. A simple differential amplifier, without emitter degeneration, is shown in Figure 5. The sum of currents (I_a) in both transistors is held constant, and the difference between them (I_o) is taken as the output. The input signal (V_i) is the difference in base voltages, and the output current as a result of base-emitter voltage (V_{be}) is:

$$I_c = I_s e^{V_{be}/V_t} \rightarrow V_{be} = V_t ln(I_c/I_s), \tag{1}$$

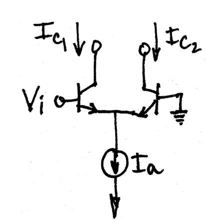

Fig. 5. BJT Differential amplifier.

where I_c is the collector current, I_s is a device specific parameter, and V_t is the "thermal voltage" (∼26mV). This gives the differential amplifier transfer function as:

$$I_a = I_{c1} + I_{c2} \quad (2)$$
$$I_o = I_{c1} - I_{c2} \quad (3)$$
$$\rightarrow I_{c1} = \frac{I_a + I_o}{2} \quad (4)$$
$$\rightarrow I_{c2} = \frac{I_a - I_o}{2} \quad (5)$$
$$V_i = V_{be1} - V_{be2} = V_t ln(I_{c1}/I_s) - V_t ln(I_{c2}/I_s) \quad (6)$$
$$V_i = V_t ln(I_{c1}/I_{c2}) = V_t ln(\frac{I_a + I_o}{I_a - I_o}) \quad (7)$$
$$\rightarrow e^{V_i/V_t} = \frac{I_a + I_o}{I_a - I_o} \quad (8)$$
$$\rightarrow (I_a - I_o)e^{V_i/V_t} = I_a + I_o \quad (9)$$
$$\rightarrow I_o(e^{V_i/V_t} + 1) = I_a(e^{V_i/V_t} - 1) \quad (10)$$
$$\Rightarrow I_o = I_a(\frac{e^{V_i/V_t} - 1}{e^{V_i/V_t} + 1}) \quad (11)$$

The Taylor series for $V_i/V_t = x$ is:

$$\frac{I_o}{I_a} = \frac{1}{2}(x - \frac{x^3}{12} + \frac{x^5}{120} - \frac{17x^7}{20160} + ...) \quad (12)$$

The Taylor series for $Y = sin(t)$ is:

$$Y = t - \frac{t^3}{6} + \frac{t^5}{120} - \frac{t^7}{5040} + ... \quad (13)$$

The differential pair matches very closely in terms of form (alternating sign, odd power series), but has the wrong coefficients to perfectly match the sine function. Since the equations only need to match for ±90° of the sine function, we can select an input amplitude to modify the coefficients. The input function to the differential amplifier is a ramp wave which increases linearly in time: $V_i/V_t = kt$, where t is time. The Taylor series then becomes:

$$\frac{I_o}{I_a} = \frac{1}{2}(kt - \frac{(kt)^3}{12} + \frac{(kt)^5}{120} - \frac{(kt)^7}{1185.88...} + ...) \quad (14)$$
$$= \frac{k}{2}(t - \frac{k^2t^3}{12} + \frac{k^4t^5}{120} - \frac{k^6t^7}{1185.88...} + ...) \quad (15)$$

By selecting $k = \sqrt{2}$, $k = 1$, or $k = 0.7857$ we can match the second, third or fourth coefficient, but, we can not match them all at once. To do this, we need more parameters, which we can add by modifying the circuit. Figure 6 shows schematics for differential pair and OTA circuits with emitter degeneration resistors (R_e). This added feedback alters the transfer function as follows:

$$V_i = (V_{be1} + I_{c1}R_e) - (V_{be2} + I_{c2}R_e) \quad (16)$$
$$= (V_{be1} - V_{be2}) + (I_{c1} - I_{c2})R_e \quad (17)$$
$$= V_t ln(I_{c1}/I_{c2}) + I_oR_e \quad (18)$$
$$= V_t ln(\frac{I_a + I_o}{I_a - I_o}) + I_oR_e \quad (19)$$
$$\rightarrow \frac{V_i}{V_t} = ln(\frac{1 + I_o/I_a}{1 - I_o/I_a}) + \frac{I_o}{I_a}\frac{I_aR_e}{V_t} \quad (20)$$

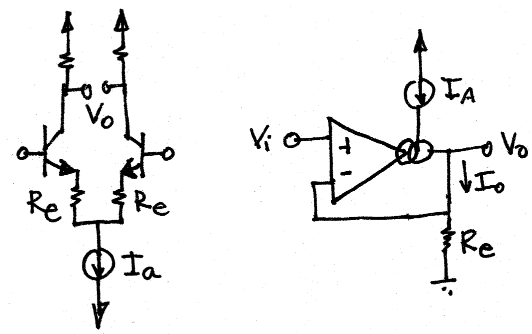

Fig. 6. Discrete and OTA based emitter degeneration sine waveshapers.

Unfortunately, this function can not be inverted to obtain I_o as a function of V_i, but we can take its Taylor series and compare it to $t = Sin^{-1}(y)$. We set $m = I_aR_e/V_t$, and again set $V_i/V_t = kt$:

$$\frac{V_i}{V_t} = 2[(\frac{m}{2} + 1)x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + ...] \quad (21)$$
$$\rightarrow t = \frac{(m+2)x}{k} + \frac{2x^3}{3k} + \frac{2x^5}{5k} + \frac{2x^7}{7k} + ... \quad (22)$$

The Taylor series of $t = Sin^{-1}(y)$ is:

$$t = y + \frac{y^3}{6} + \frac{3y^5}{40} + \frac{5y^7}{112} + ... \quad (23)$$

But, in this case the magnitude of the output equation does matter, as time must be equal between the two functions, so the coefficients must be exactly matched. This gives $k = 4$, $m = 2$ for matching the first and second coefficients, or $k = 16/3$, $m = 10/3$ for matching the first and third coefficients.

Again, we can only match two of the coefficients, but the addition of the m parameter allows a finer control over the first coefficient (the initial slope), and therefore the circuit can be tuned for lower distortion. Robert Meyer explored this topology in "The Differential Pair as a Triangle-Sine Wave Converter', 1976, and found a lower bound on distortion of 0.2%, with $m = 2.5$ and $k = 4.2$. Meyer defines the term V_M/V_t, which is optimized to 6.6. This is equal to our kt at $t_{max} = \pi/2$, so $k = 4.2$.

To improve upon this, we will need to decouple the parameters which control the upper coefficients. Adding a bit of the original signal (cusp canceling) allows us to do this. In more physical terms, the original differential amplifier has too much curvature at the start of its slope, and never really achieves a zero slope function, which is required at the top of a sine wave. The emitter degeneration resistor helps straighten up the slope at the beginning, but still doesn't allow for a zero slope condition at reasonable input levels. By subtracting some of the input signal, we both straighten up the input, and create a zero slope point as the input becomes larger than the output. The signal even reverses, giving an almost ±180° of usable range.

An OTA implementation of cusp canceling is shown in Figure 7. The output is simply a mix of the differential pair output and the original signal. The difficulty here is to pick the

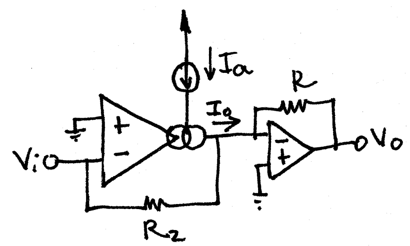

Fig. 7. OTA based sine shaper with cusp canceling.

relative amplitude of these signals to minimize distortion. By looking at the Taylor series, we can get an idea of the optimal ratios.

$$\frac{V_i}{V_t} = k_1 t \tag{24}$$

$$\frac{V_t}{I_a R_2} = \frac{k_2}{k_1} \tag{25}$$

$$V_o = R[I_o - \frac{V_i}{R_2}] = R[I_o - \frac{V_t}{R_2}\frac{V_i}{V_t}] \tag{26}$$

$$= RI_a[(\frac{e^{V_i/V_t} - 1}{e^{V_i/V_t} + 1}) - \frac{V_t}{I_a R_2}\frac{V_i}{V_t}] \tag{27}$$

$$= RI_a[(\frac{e^{k_1 t} - 1}{e^{k_1 t} + 1}) - \frac{k_2}{k_1}k_1 t] \tag{28}$$

$$\begin{aligned} &= \frac{I_a R}{2}(k_1 - 2k_2)[t - \frac{k_1^3 t^3}{12(k_1 - 2k_2)} \\ &+ \frac{k_1^5 t^5}{120(k_1 - 2k_2)} - \frac{17 k_1^7 t^7}{20160(k_1 - 2k_2)} + ...] \end{aligned} \tag{29}$$

Since absolute amplitude doesn't matter, we can ignore the $I_a R/2$ at the beginning, and match the coefficients of Equations 13 and 29. Because of the normalization done by dividing out the amplitude, the first coefficient already matches, so this gives the second and third as:

$$\frac{k_1^3}{12(k_1 - 2k_2)} = \frac{1}{6} \tag{30}$$

$$\frac{k_1^5}{120(k_1 - 2k_2)} = \frac{1}{120} \tag{31}$$

$$\rightarrow k_1 = \frac{\sqrt{2}}{2} \approx 0.7071 \tag{32}$$

$$\rightarrow k_2 = \frac{3\sqrt{2}}{16} \approx .2652 \tag{33}$$

Because of the mismatch in the remaining coefficients, better performance can be obtained by slightly shifting these results. Figure 8 shows the theoretical THD of the circuit for varying k_1 and k_2 parameters. These were simulated with a computer over the first $\pm 90°$, taking the FFT output and dividing the geometric sum of the harmonics by the fundamental. The optimal parameters were found to be $k_1 = 0.7476$ and $k_2 = 0.270065$ with a THD of -83.37dB (0.0068%).

IV. Results

All three variants of the OTA based sine waveshapers were built and tested for distortion. The circuits used are shown in

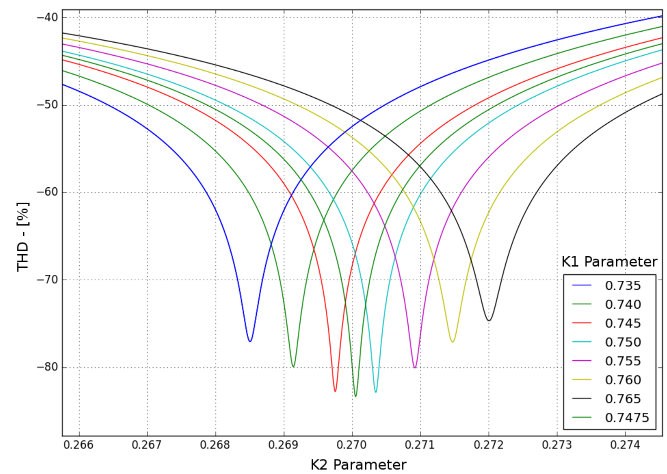

Fig. 8. Computer simulation of OTA sine waveshaper circuit performance for various k_1 and k_2 values.

Figures 9–10, and they performed quite close to the predicted levels. The circuits were driven with a triangle wave of variable amplitude from an HP-33120A signal generator, and the output harmonics were measured with an HP-3561A dynamic signal analyzer. The drive signal was 1.3kHz, and the harmonic content was summed over a 20kHz bandwidth (up to the 13^{th} harmonic).

Every topology tested was able to null all of the even harmonics out of the signal. The basic OTA circuit was able to null the 3^{rd} harmonic, but the remaining harmonics ranged from -38dB at the 5^{th} down to -54dB at the 13^{th}. This gives a total of -37dB THD (1.4%). The emitter degeneration case performed much better, with both the 3^{rd} and 5^{th} harmonics nulling out, and the others decaying from -54dB at the 7^{th} down to -60dB at the 13^{th}. This gives a total of -51dB THD (0.28%), which is quite close to Meyer's result of "about 0.2%". The cusp canceling method performed the best, with all harmonics being pushed into the noise floor of the signal analyzer. Small peaks could be measured for the 2^{nd}, 3^{rd}, and 5^{th} harmonics around -80dB, but the remainder were unmeasurable (<-85dB). This gives a total of -74dB (0.02%), assuming all remaining harmonics were at -85dB. If the 2^{nd} harmonic were better trimmed out, and the remaining harmonics are assumed to be at a negligible level, those figures become -77dB THD (0.014%). It's possible this figure can be pushed even lower by reducing the input impedance to the OTA, thereby eliminating the OTA base current errors. A

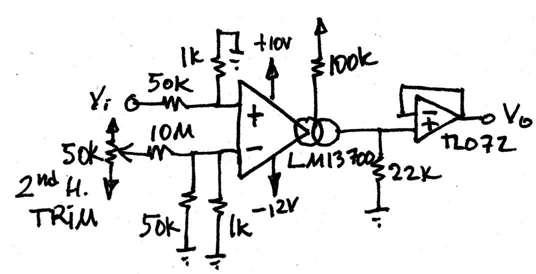

Fig. 9. Tested circuit for basic, over-driven OTA sine waveshaper.

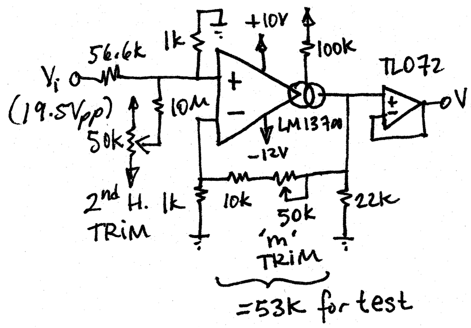

Fig. 10. Tested circuit for OTA sine waveshaper with feedback (emitter degeneration).

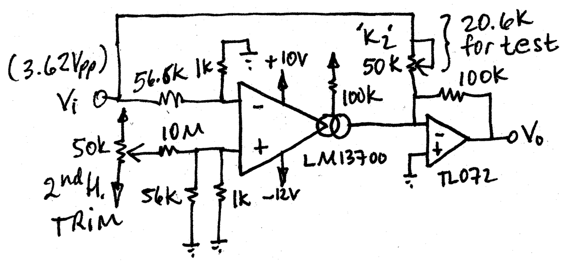

Fig. 11. Tested circuit for OTA sine waveshaper with cusp canceling.

second circuit was built with 36Ω resistors replacing the 1kΩ resistors in Figure 11, and it had -80dB THD (0.01%).

The cusp canceling distortion figures may seem too good to be true, and in some ways they are. The first thing to note, is that the OTA gain equation is severely dependent upon V_t, since $V_t = kT/q$ (where k is Boltzmann's constant, T is temperature, and q is the charge of an electron). So some form of compensation will be required. Secondly, as shown in Figure 8, the THD null is extremely sharp. The full range shown in the figure is over ±1% change in k_1 and k_2. This means that a 1% change in component values, or transconductance of the OTA, could increase the THD to -50dB (0.3%). Granted, this is still better than many of the other options at their best, not accounting for drifts they may have. A quick test with a hot air gun, raising the temperature ∼ 30°C, shifted all odd harmonics up to -50dB (-42dB THD (0.77%)).

The second issue with the circuit, is how difficult it is to tune. As can be seen in Figure 8, both k_1 and k_2 interact with each other, so iterative tuning is required. This can be accomplished by making a small change in one parameter, and then re-nulling the other. After this, the first parameter is changed again in the same direction as before (if it was lowered, it is lowered once again), and the second parameter is re-nulled again. The THD should fall with each adjustment, and adjustment stops when THD begins to rise.

V. Conclusions

An OTA based sine shaper is a relatively inexpensive and well performing circuit, if cusp canceling is used. For best results it should be built with temperature compensation, and a simple thermistor is good enough, as any extra complexity would make other options more favorable. If a thermistor is employed, it should replace the 1kΩ resistor at the inverting terminal of the OTA shown in Figure 11. This will allow k_1 to track with temperature, but keep k_2 constant, as it is not affected by V_t. Other modifications to this circuit would include smaller value trimmers with larger resistors in series with them, to give a smaller tuning range. The input to the entire circuit also needs an attenuator of some sort, as the amplitude was adjusted via the function generator for this work. Finally, for even more temperature stability, the current source for the OTA (I_a) could be fixed with an opamp current source, as the voltage at the OTA bias input pin varies with temperature. With large voltage rails, this variation becomes less of an issue.

The values of $k_1 = 0.7476$ and $k_2 = 0.270065$ can be used to tailor the OTA sine shaper circuit for a particular application. From equations 24 and 25, we can find attenuation ratios with respect to the single sided, maximum input voltage, V_{max}, which occurs at $t_{max} = \pi/2$.

$$V_t = 26mV \text{ @ } 28°\text{C} \tag{34}$$

$$\frac{V_i}{V_t} = k_1 t \rightarrow V_i = k_1 V_t t \tag{35}$$

$$\Rightarrow V_{max} = \frac{k_1 V_t \pi}{2} = 30.53mV \tag{36}$$

This represents the maximum voltage applied to the inverting terminal of the OTA, not the circuit input voltage. To obtain the circuit input voltage, this V_{max} value must be multiplied by the resistor divider ratio. For example, in the circuit of Figure 11, The input voltage is 3.62V$_{pp}$ = 1.81V$_p$, which gives V_{max} = 1.81V/57.6 = 31mV, since the resistor divider is 1kΩ and 56.6kΩ.

Similarly, k_2 sets the maximum current (I_{max}) flowing into the inverting terminal of the opamp, as follows:

$$I_{max} = \frac{V_{max}}{R_2}, \quad V_{max} = \frac{k_1 V_t \pi}{2} \tag{37}$$

$$\frac{V_t}{I_a R_2} = \frac{k_2}{k_1} \rightarrow R_2 = \frac{k_1 V_t}{k_2 I_a} \tag{38}$$

$$\Rightarrow I_{max} = \frac{k_2 V_{max}}{k_1 V_t} I_a = \frac{k_2 \pi}{2} I_a \approx 0.42422 \cdot I_a \tag{39}$$

In the example circuit, $I_a \approx$ 20.6V/100kΩ = 206μA, which gives I_{max} = 87.4μA. With an input voltage of 1.81V$_p$, R_s = 1.81V/87.4μA= 20.7kΩ. So the theory matches the experimental quite well, with the slight mismatch of V_{max} most likely due to the approximation of V_t.

For improved performance over the OTA, the $x/(1 + x^2)$ circuits of Figure 3 should have better temperature stability. The Gilbert shaper of Figure 4 adds a fair bit of complexity, but also opens up possibilities the others do not. It can be used as a wavefolder, or to add Phase Modulation (PM) to any VCO merely by adding the PM signal to the VCO output, which would give it through-zero capability.